Recitation 12/1 Solutions

1. a) the prize is behind door 3 and you decide to open door 1.

<u>Possible Outcomes</u>:

	you choose	host opens	switch/stay
lose	1	2	1 (stay)
win	1	2	3 (switch)

* <u>note</u>: this is different from previous counting problems b/c these 2 outcomes do NOT occur with ~~equal~~ equal probability (equal prob. required for counting problems)
ie. we do not know the chance that the player will stay/switch

b) stated: you flip a coin to decide whether to stay or switch

We can now assign probabilities to the 2 outcomes in (a), and end up with a probability model (table of x and $p(x)$).

x	$p(x)$	
lose (stay)	$\frac{1}{2}$	coin flip
win (switch)	$\frac{1}{2}$	

Alternatively, you can rewrite this table (prob. model) as:

x	lose	win
$p(x)$	$\frac{1}{2}$	$\frac{1}{2}$

which is the layout used in previous recitations. Both are the same.

Therefore $P(\text{win}) = \frac{1}{2}$ due to coin flip.

2. a) Based on the game mechanics, there are a total of 12 outcomes (if we do not count the stay/switch part).

Game Mechanics:

① you choose a door

② host opens a door that does not hold the prize

Outcomes:

prize location	you choose	host opens
1	1	2
1	1	3
1	2	3
1	3	2
2	1	3
2	2	1
2	2	3
2	3	1
3	1	2
3	2	1
3	3	1
3	3	2

*note: just like in Q1, these outcomes do not occur with the same equal probability (we don't know the prob. of each outcome)

ie. no probability model b/c we don't know probability of outcomes

b) stated: · player will choose a door with equal prob.
· adopt a policy of always stay with initial pick (KEEP policy)

Then the player has a $\frac{1}{3}$ chance of picking the door with the prize.

$P(\text{win}) = \frac{1}{3}$

c) We can illustrate (b) with a prob. model.

stated: prize behind door 1

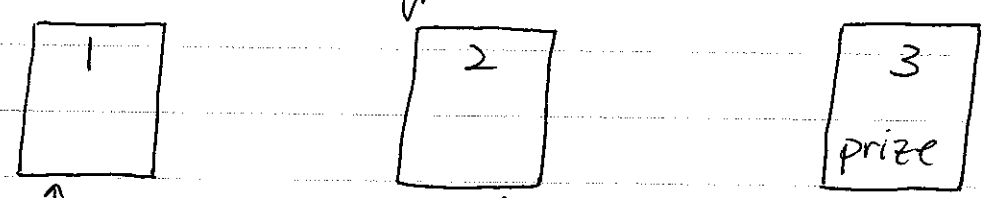
x	1	2	3
$p(x)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

X = door (door #)

$P(\text{win}) = P(1) = \frac{1}{3}$

d) stated: player will always switch (Switch Policy)

Let's look at a hypothetical case:



① you pick ② host opens

If you stay, you will lose with prob. $\frac{1}{3}$ as we found in (b),(c). Since you only have 2 choices left:

· stay with door 1

· switch to door 3

Then by the idea of complements (opposites), you will win with prob. $1 - \frac{1}{3} = \frac{2}{3}$.

Therefore, $P(\text{win}) = \frac{2}{3}$.

e) stated : win = \$10000 , lose = \$0

For the KEEP policy : (X = amount won)

x	10000 (win)	0 (lose)
$p(x)$	$\frac{1}{3}$	$\frac{2}{3}$

$$E(X) = \sum_{\text{all } x} x\,p(x) = 10000\left(\tfrac{1}{3}\right) + 0\left(\tfrac{2}{3}\right)$$
$$= 3333$$

$$E(X^2) = \sum_{\text{all } x} x^2 p(x) = 10000^2\left(\tfrac{1}{3}\right) + 0^2\left(\tfrac{2}{3}\right)$$
$$= 33333333$$

$$Var(X) = E(X^2) - (E(X))^2$$
$$= 33333333 - 3333^2$$
$$= 22222222$$

$$SD(X) = \sqrt{Var(X)}$$
$$= \sqrt{22222222}$$
$$= 4714$$

f) For the Switch policy :

y = amount won

y	10000 (win)	0 (lose)
$p(y)$	$\frac{2}{3}$	$\frac{1}{3}$

$$E(y) = 6667$$
$$E(y^2) = 66666667$$
$$Var(y) = E(y^2) - (E(y))^2 = 66666667 - 6667^2$$
$$= 22222222$$
$$SD(y) = \sqrt{Var(y)} = \sqrt{22222222} = 4714$$

3. stated: a random variable W has $E(W) = 41.7$, $SD(W) = 17.2$

a) stated: do Not know distribution of W (ie. no prob. model)

We therefore find $E(3W+2)$ using property of expectations:

* <u>Property</u>: $E(aX + bY + c) = aE(X) + bE(Y) + c$

$$E(3W+2) = 3E(W) + 2$$
$$= 3(41.7) + 2$$
$$= 127.1$$

Find $Var(3W+2)$.

* use property of Variances: $Var(aX + b) = a^2 Var(X)$

<u>note</u>: if you have more than 1 variable (eg. X & Y), then you need to satisfy the condition "X & Y independent" before applying this property

$$Var(3W+2) = 3^2 Var(W)$$
$$= 3^2 (17.2)^2$$
$$= 2662.56$$

Find $SD(3W+2)$.

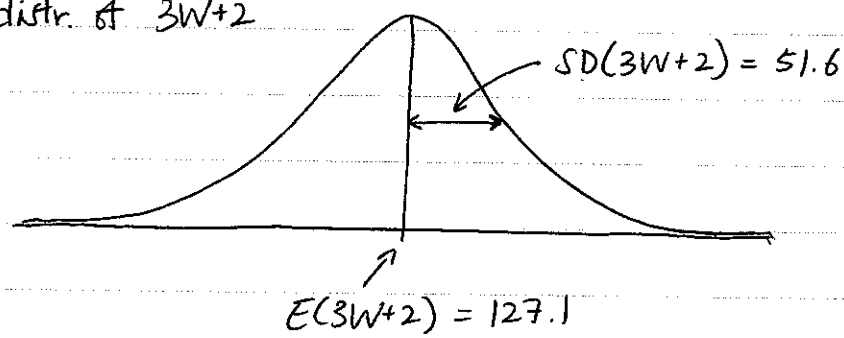
$$SD(3W+2) = \sqrt{Var(3W+2)}$$
$$= \sqrt{2662.56}$$
$$= 51.6$$

b) stated : W is normally distributed

$\Rightarrow$ (result) then $3W+2$ is ALSO normally distributed

We can draw the bell curve for $3W+2$. This is useful for finding areas under the bell curve (eg.) find $P((3W+2) > 150)$).
We will see an example in Q4.

distr. of $3W+2$



c) stated : do NOT know distr. of W

If we have 100 (doesn't have to be 100) random variables $W_1, W_2, \ldots, W_{99}, W_{100}$, where all of them have the "same distribution" as W (this is the random variable initially stated in Q3) then :

$$sum = W_1 + W_2 + \cdots + W_{100}$$

"(result) sum is normally distributed."

"same distribution" means they have the same expected value and same SD.

ie. $E(W_1) = E(W_2) = \cdots = E(W_{100}) \overset{\text{same distr.}}{=} E(W)$

$SD(W_1) = \cdots = SD(W_{100}) = SD(W)$

$$E(\text{sum}) = E(W_1 + \cdots + W_{100})$$
$$= E(W_1) + \cdots + E(W_{100}) \quad \text{by property of expectations}$$
$$= 41.7 + \cdots + 41.7$$
$$= 41.7 \times 100$$
$$= 4170$$

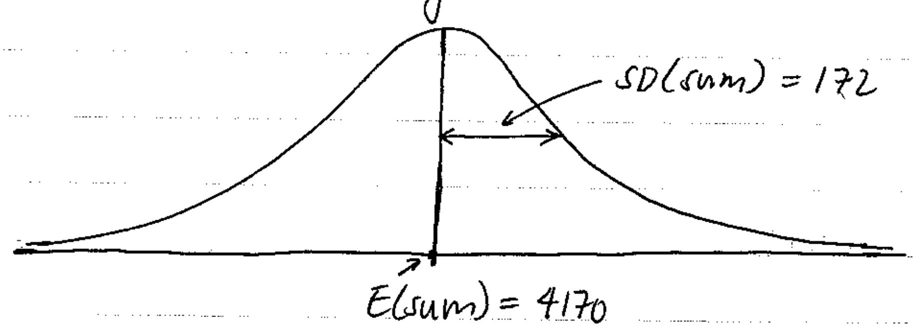
$$Var(\text{sum}) = Var(W_1 + \cdots + W_{100})$$

* use property of variances :
if X and Y are independent, then
$$Var(X+Y) = Var(X) + Var(Y)$$

$$Var(\text{sum}) = Var(W_1) + \cdots + Var(W_{100})$$
$$= 17.2^2 + \cdots + 17.2^2$$
$$= 17.2^2 \times 100$$
$$= 29584$$

$$SD(\text{sum}) = \sqrt{Var(\text{sum})}$$
$$= \sqrt{29584}$$
$$= 172$$

Since sum is normally distr. :



4. stated: $E(X) = -0.035$ $\quad E(Y) = -0.024$

$SD(X) = 3.65$ $\quad SD(Y) = 4.11$

a) stated: · John will play 500 games, 100 rounds of game X and 400 rounds of game Y

· all 500 games are independent

notation: $X_1, \dots, X_{100}$ represent the amount won in each of the 100 X games respectively

$Y_1, \dots, Y_{400}$ represent the amount won in each of the 400 Y games played respectively

Then $net = X_1 + \cdots + X_{100} + Y_1 + \cdots + Y_{400}$ is the net amount of money John won in the 500 games he played.

$$E(net) = E(X_1 + \cdots + X_{100} + Y_1 + \cdots + Y_{400})$$

(property of expectations) $= E(X_1) + \cdots + E(X_{100}) + E(Y_1) + \cdots + E(Y_{400})$

$$= (-0.035) + \cdots + (-0.035) + (-0.024) + \cdots + (-0.024)$$

$$= (-0.035) \times 100 + (-0.024) \times 400$$

$$= (-3.5) + (-9.6)$$

$$= -13.1$$

$$Var(net) = Var(X_1 + \cdots + X_{100} + Y_1 + \cdots + Y_{400})$$

$$= Var(X_1) + \cdots + Var(X_{100}) + Var(Y_1) + \cdots + Var(Y_{400})$$

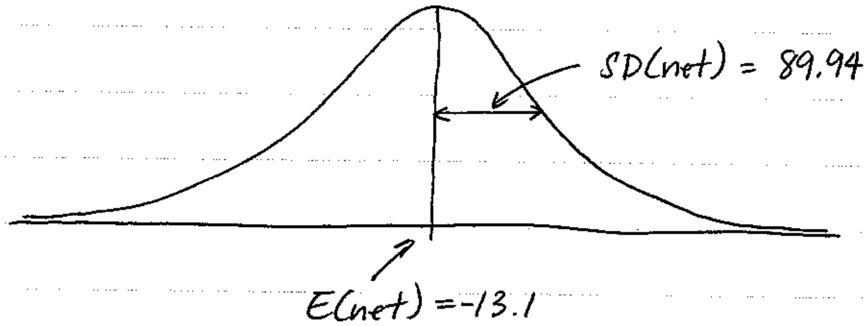
(by applying the property of variances since all 500 games are independent)

$$= 3.65^2 \times 100 + 4.11^2 \times 400$$

$$= 8089.09$$

$$SD(net) = \sqrt{Var(net)}$$
$$= \sqrt{8089.09}$$
$$= 89.94$$

net is normally distr. (same idea as Q3 (c))



b) stated: John plays each game once but puts 100 times the money on game X and 400 times the money on game Y

$$E(100X + 400Y) = 100E(X) + 400E(Y)$$ prop. of expectations
$$= 100(-0.035) + 400(-0.024)$$
$$= -13.1$$

$$Var(100X + 400Y) = 100^2 Var(X) + 400^2 Var(Y)$$
(by property of variances w/ condition X & Y independent)
$$= 100^2(3.65)^2 + 400^2(4.11)^2$$
$$= 2835961$$

$$SD(100X + 400Y) = \sqrt{Var(100X + 400Y)}$$
$$= \sqrt{2835961}$$
$$= 1684.03$$

In both cases, we would expect John to lose 13.1 dollars. However, the fluctuation in winnings is much higher in the second case (SD was 1684). In other words, the second case is much riskier as John could win ~~bet~~ or lose much more than in case 1.

c) In (a) we found $E(net) = -13.1$

$SD(net) = 89.94$.

Since net follows a normal distr. :

P(John's total winnings are positive)

$= P(net > 0)$

$= P\left(Z > \frac{0-(-13.1)}{89.94}\right)$

$= P(Z > 0.146)$

$= 0.442$ or 44.2%

P(John's total winnings are below -200)

$= P(net < -200)$

$= P\left(Z < \frac{-200 - (-13.1)}{89.94}\right)$

$= P(Z < -2.078)$

$= 0.019$ or 1.9%

5. X = net winnings

x	999	-1
$p(x)$	0.000965	0.999035

a) $E(X) = \sum_{\text{all } x} x\,p(x) = 999(0.000965) + (-1)(0.999035)$

$= -0.035$

$Var(X) = E(X^2) - (E(X))^2$ so find $E(X^2)$ first

$E(X^2) = \sum_{\text{all } x} x^2 p(x) = 999^2(0.000965) + (-1)^2(0.999035)$

$= 964.07$

$Var(X) = 964.07 - (-0.035)^2$

$= 964.069$

$SD(X) = \sqrt{Var(X)} = \sqrt{964.069} = 31.05$

b) question should read "betting \$400 in one round"

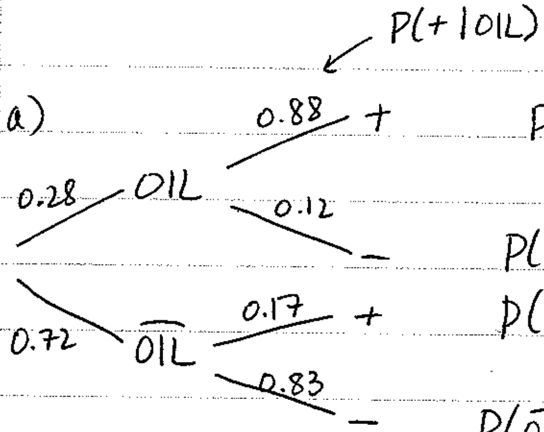
John stands to win a max of $400 \times 999 = 399600$. The prob. of this is the same as before $= 0.000965$.

c) $E(400X) = 400E(X) = 400(-0.035) = -14$

$Var(400X) = 400^2 Var(X) = 400^2(964.069) = 154251040$

$SD(400X) = \sqrt{Var(400X)} = \sqrt{154251040} = 12419.78$

6. a)



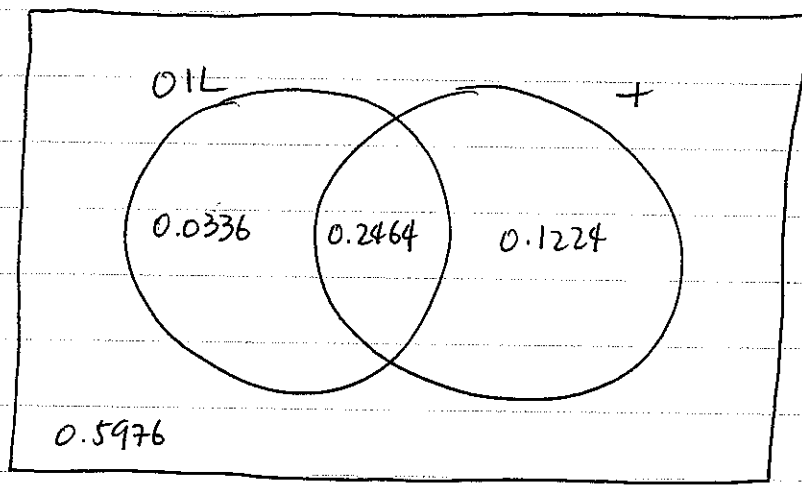
$P(\text{OIL} \cap +) = 0.28(0.88) = 0.2464$

$P(\text{OIL} \cap -) = 0.28(0.12) = 0.0336$

$P(\overline{\text{OIL}} \cap +) = 0.72(0.17) = 0.1224$

$P(\overline{\text{OIL}} \cap -) = 0.72(0.83) = 0.5976$

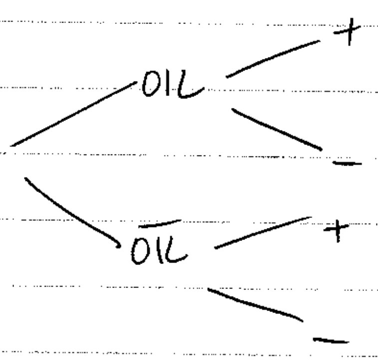
b)



stated: cost 80 to test, cost 900 to drill, earn 3000 when oil is found

c) adopt policy: only drill if test is positive

X = net return



x	$p(x)$
$-80-900+3000 = 2020$	0.2464
-80	0.0336
$-80-900 = -980$	0.1224
-80	0.5976

d) the prob. model from (c) is:

x	2020	−80	−980	−80
$p(x)$	0.2464	0.0336	0.1224	0.5976

$E(X) = 2020(0.2464) + (-80)(0.0336) + (-980)(0.1224) + (-80)(0.5976)$

$= 327.28$

You would expect to earn 327.28 under this policy: only drill if test is positive.

e) policy: just drill
(I'm going to do this question differently from recitation.)
X = net return

when there is oil: net return is $-900 + 3000 = 2100$
when there is no oil: net return is -900

x	2100 (oil)	-900 (no oil)
$p(x)$	0.28	0.72

$$E(X) = 2100(0.28) + (-900)(0.72)$$
$$= -60$$

You would expect to make a loss of 60 under this policy.